Introduction

Motivation

Internet is now in widespread use for storing learning resources and delivering on-line courses. These materials can be used as a substantially complete set of materials in an on-line course. However, rapid growth of on-line learning resources has caused a problem to students that they find it getting more difficult to decide which learning resources best meets their needs. Therefore, personalized learning has become an important research issue because it can deliver what is needed at the right time to improve students' learning achievements (Zaiane, 2001).

There have been many research efforts regarding the issues of personalized learning on the web (Berlanga & Garcia, 2004; Papanikolaou & Grigoriadou, 2003; Sampson, Karagiannidis, & Cardinali, 2002). To produce effective personalized learning, several types of knowledge are required (Brusilovsky & Vassileva, 2003). The first is the domain knowledge that depicts the concepts and their interrelations in a specific target domain. The second is the student modeling knowledge that describes a student's knowledge level, learning style and so on. The last is the resource knowledge that describes educational characteristics of learning resources. A personalized learning system should exploit the aforementioned knowledge to attend the needs of each student as best as possible. In particular, to deliver appropriate materials to students, a student model that can accurately predict a student's strength and/or weakness of concepts/skills will be helpful to identify the specific needs of the individual student.

As the aforementioned purpose is concerned, Diagnostic Test (DT) is a potential tool to diagnose a student's strength and weakness of concepts/skills of a course. The diagnostic test is important because it exposes information about a student's learning progression during a learning process. Through the results of diagnostic tests, an e-learning system could gain immediate feedbacks about how well students are learning particular concepts/skills. Based on these feedbacks, the system can adjust the difficulty of the content materials responsively to supplement their learning of the corresponding concepts/skills.

The Problem

A simple representation, called *Q*-matrix (or item-concept table) (Tatsuoka, 1990), is often used to depict the relationship between items and target concepts/skills. In a *Q*-matrix, the row indices are items and the column indices are concepts/skills. Furthermore, in a *Q*-matrix, an entry $q(i, j) = 1$ means the piece of concept/skill j is required to solve the item i, and $q(i, j) = 0$ means the concept/skill j is not required for solving item i. As a consequence, the research problem can be stated as follows:

Given a set of K concepts/skills, a test containing J items, a $J \times K$ *Q*-matrix, and N student item responses, each of which consists of a J-long binary vector indicating which test items were answered correctly or incorrectly, the problem considered in this research is to output a K-long real-numbered vector indicating the mastery degree of each concept/skill for each student.

Study on the aforementioned problem has been attracting the attention of researchers in recent years. However, while many diagnostic methods and algorithms have been proposed (Cheng, Lin, Chen, & Heh, 2005; Hsu, Chang, Chang, Jehng, & Heh, 2002; Hwang, Hsiao, & Tseng, 2003; Kuo, Chang, Dong, & Heh, 2002; VanLehn, Niu, Siler, & Gertner, 1998), most of them are heuristic in essence and are in lack of a solid theoretical foundation.

In the literature, it was generally assumed that when a student answered some item correctly, all the concepts/skills involved in that item would be assumed mastered (Birenbaum & Tatsuoka, 1993; Brown & Burton, 1978; Sleeman, 1984). However, this assumption might be problematic in some situations. For example, consider the *Q*-matrix with five concepts/skills and the item response sample shown in Figure 1. In this case it is hard to explain why a student answered correctly item I_1 and item IJ, but failed item I_2 (as C_1, C_2 and C_3 were supposed to be mastered due to the correct responses of item I_1 and IJ). Several heuristic (Cheng et al., 2005; Hwang et al., 2003) and probabilistic approaches (Birenbaum & Tatsuoka, 1993; Tatsuoka, 1990) have been proposed to deal with the aforementioned problem. This paper proposes an alternative approach that is based on an extension of the dichotomous IRT model to deal with this problem.