

Introduction

Although MKT and its domains have been the subject of much research and discussion, and of many educational reform efforts in recent years (e.g., Ball, Thames, & Phelps, 2008; Bell, Wilson, Higgins, & McCoach, 2010; Hill, 2010; Kersting, Givvin, Sotelo, & Stigler, 2010), little is known about the general conception and its implementation at the high school level. Most studies of MKT have focused on elementary mathematics instruction. Furthermore, it seems unlikely that high school teachers would lack common content knowledge (CCK). As a result, it may seem more suitable for researchers to investigate other kinds of knowledge (i.e., MKT) required for effective teaching at the high school level.

There is currently no well-described method to measure a high school teacher's MKT and few research has been done to create items for assessing high school teachers' MKT. This paper details an initial effort to develop several items that capture high school mathematics teachers' MKT. In particular, the report documents how the representative item was developed from beginning to end. An account of this work should be helpful and useful to those who want to conceptualize, identify and measure MKT at the high school level. Furthermore, this work is an important precursor to the design and implementation of large-scale studies to assess high school mathematics teachers' MKT. Finally, the long-term intention for this work is to help high school mathematics teachers to improve their MKT, which will benefit in students' learning.

The purpose of this study is to describe the process of developing the items to access high school teachers' MKT. It should be noted that our interest in this study is not about how high school teachers perform on the MKT items but in reporting the process (method) to create items to measure high school teachers' MKT. We asked:

1. What is the process of developing high school MKT items?
2. What principles are useful in creating high school MKT items?
3. What items are usable in capturing high school teachers' MKT?

Literature Review

Mathematics Knowledge for Teaching

The notion of MKT refers to the mathematics knowledge needed to carry out the work of teaching mathematics beyond what can be seen in the tables of contents of school textbooks or in

curriculum frameworks (Ball et al., 2008). MKT has traditionally been divided into six domains.

The first domain, CCK, refers to mathematics knowledge and the skills used in an applied, rather than a teaching, setting. CCK includes simply calculating an answer, knowing the material to be taught, or, more generally, correctly solving a problem (Ball et al., 2009). Examples of questions that can be used to measure teacher CCK include “What is a number that lies between 1.1 and 1.11?”, “Is a square a rectangle?” “Is $0/7$ equal to 0?” and “Are diagonals of a parallelogram necessarily perpendicular?” (Ball et al., 2008). The second domain, specialized content knowledge (SCK), is specialized to the work of teaching and only teachers need to know it (Hill, Sleep, Lewis, & Ball, 2007). SCK is the mathematics knowledge and skill unique to teaching and requires knowledge beyond that being taught to students. Herbst and Kosko (2012) defined SCK as mathematics knowledge used particularly to accomplish the tasks of teaching. Such knowledge encompasses things that teachers routinely do in the classroom, such as “presenting mathematical ideas,” “responding to students’ why questions” or “finding an example to make a specific mathematical point” (Ball et al., 2008). Horizon content knowledge (HCK) is an understanding of the broader set of mathematics ideas to which a particular idea connects (Ball et al., 2008). HCK is the sort of understanding that gives teachers peripheral vision for where they are and where their pupils are headed, so they are conscious of the consequences of how ideas are represented or of the later development that is enabled (Ball et al., 2009). Elaborating on Ball et al.’s (2008) idea of HCK, Jakobsen, Thames, Ribeiro, and Delaney (2012, p. 4642) further characterize HCK as enabling teachers “to make judgments about the importance of particular ideas or questions, and to treat the discipline with integrity, all resources for balancing the fundamental task of connecting learners to a vast and highly developed field”. In this study, we define HCK as a body of knowledge that allows a teacher to recognize most core (fundamental) concepts in teaching problems.

The other domains of MKT represent strands associated with PCK and contain knowledge of content and students (KCS), knowledge of content and teaching (KCT) and knowledge of content and curriculum (KCC). These comprise the special amalgams that are deeply embedded in the work of teaching what makes a topic difficult for pupils; the ways in which learners tend to develop understanding of a particular idea; and, ways to sequence and structure the development of a mathematics topic, including representations likely to help pupils learn (Ball et al., 2009; Ball et al., 2008; Hill, Ball, & Schilling, 2008).

KCS is very closely related to student thinking. It is knowledge of common student conceptions and misconceptions about particular mathematical concept (Ball et al., 2008). For example, “when assigning a task, teachers need to anticipate what students are likely to do with it and whether they